Horn Torus Models for the Riemann Sphere and Division by Zero

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Abstract: In this paper, we will introduce a beautiful horn torus model by Puha and Däumler for the Riemann sphere in complex analysis attaching the zero point and the point at infinity. Surprisingly enough, we can introduce analytical structure of conformality to the model.

Key Words: Infinity, discontinuous, point at infinity, stereographic projection, Riemann sphere, horn torus, Laurent expansion, conformal mapping, division by zero.

Mathematics Subject Classification (2010): 30C25, 30F10

Introduction

In this paper we introduce a new horn torus model for the classical Riemann sphere from the viewpoint of the division by zero. In the model, the zero point and the point at infinity are attaching and surprisingly enough, we can introduce analytical structure of conformality in the horn torus. This model seems to be fundamental and important for us.

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In Section 1, we introduce the horn torus model with the Puha mapping that gives one to one mapping from the extended complex plane onto a horn torus through the stereographic projection mapping from the Riemann sphere onto the extended complex plane and in Section 2, we state simple properties of the Puha mapping.

In Section 3, we will introduce a conformal mapping from the extended complex plane onto the horn torus by modifying the Puha mapping. This introduction by Däumler is geometrical and delicate, and the mapping will be very complicated. Therefore, we will give a complete confirmation for his conformal mapping analytically in Section 5. In Section 4, we give the elementary properties of the Däumler conformal mapping. In Section 5, we will simply refer to the division by zero in connection with the horn torus model.

1   Horn torus model

We will consider the three circles represented by

$$\xi^2 + \left(\zeta - \frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2,$$

$$\left(\xi - \frac{1}{4}\right)^2 + \left(\zeta - \frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2, \tag{1.1}$$

and

$$\left(\xi + \frac{1}{4}\right)^2 + \left(\zeta - \frac{1}{2}\right)^2 = \left(\frac{1}{4}\right)^2.$$

By rotation on the space (ξ, η, ζ) on the (x, y) plane as in $\xi = x, \eta = y$ around ζ axis, we will consider the sphere with $1/2$ radius as the Riemann sphere and the horn torus made in the sphere.

The stereographic projection mapping from (x, y) plane to the Riemann sphere is given by

$$\xi = \frac{x}{x^2 + y^2 + 1},$$

$$\eta = \frac{y}{x^2 + y^2 + 1},$$

and

$$\zeta = \frac{x^2 + y^2}{x^2 + y^2 + 1}.$$

Of course,
$$\xi^2 + \eta^2 = \zeta(1 - \zeta),$$
and
$$x = \frac{\xi}{1 - \zeta}, y = \frac{\eta}{1 - \zeta}. \tag{1.2}$$

The mapping from (x, y) plane to the horn torus is given by
$$\xi = \frac{2x\sqrt{x^2 + y^2}}{(x^2 + y^2 + 1)^2},$$

$$\eta = \frac{2y\sqrt{x^2 + y^2}}{(x^2 + y^2 + 1)^2},$$
and
$$\zeta = \frac{(x^2 + y^2 - 1)\sqrt{x^2 + y^2}}{(x^2 + y^2 + 1)^2} + \frac{1}{2}.$$

This Puha mapping has a simple and beautiful geometrical correspondence. At first for the plane we consider the stereographic mapping to the Riemann sphere and next, we consider the common point of the line connecting the point and the center $(0,0,1/2)$ and the horn torus. This is the desired point on the horn torus for the plane point.

Indeed, we denote tentatively a point with (ξ_1, η_1, ζ_1) on the horn torus. Then, we have, from the relation between a point (ξ, η, ζ) on the Riemann sphere and the correspondent point (ξ_1, η_1, ζ_1) on the horn torus
$$\xi_1 = \xi\frac{\zeta_1 - 1/2}{\zeta - 1/2}, \quad \eta_1 = \eta\frac{\zeta_1 - 1/2}{\zeta - 1/2}.$$

We set
$$\sqrt{\xi^2 + \eta^2} = t, \quad \zeta_1 - \frac{1}{2} = n, \quad \zeta - \frac{1}{2} = m.$$
Then, since the point (ξ_1, η_1, ζ_1) is on the horn torus, from the identity
$$n^2 + \left(n\frac{t}{m} - \frac{1}{4}\right)^2 = \frac{1}{16},$$
we obtain the identity
$$n = \frac{mt}{2(m^2 + t^2)}.$$

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Therefore, we obtain

$$\xi_1 = \frac{\xi}{2} \frac{\sqrt{\xi^2 + \eta^2}}{(\zeta - 1/2)^2 + \xi^2 + \eta^2},$$

$$\eta_1 = \frac{\eta}{2} \frac{\sqrt{\xi^2 + \eta^2}}{(\zeta - 1/2)^2 + \xi^2 + \eta^2}$$

and

$$\zeta_1 = \frac{1}{2} \frac{(\zeta - 1/2)\sqrt{\xi^2 + \eta^2}}{(\zeta - 1/2)^2 + \xi^2 + \eta^2} + \frac{1}{2}.$$

Hence, in terms of (x, y), we have the desired results.

The inversion is given by

$$x = \xi \left(\xi^2 + \eta^2 + \left(\zeta - \frac{1}{2} \right)^2 - \zeta + \frac{1}{2} \right)^{(-1/2)} \tag{1.3}$$

and

$$y = \eta \left(\xi^2 + \eta^2 + \left(\zeta - \frac{1}{2} \right)^2 - \zeta + \frac{1}{2} \right)^{(-1/2)}. \tag{1.4}$$

For the properties of horn torus with physical applications, see [2].

2 Properties of horn torus model

At first, the model shows the strong symmetry of the domains $\{|z| < 1, z = x + iy\}$ and $\{|z| > 1\}$ and they correspond to the lower part and the upper part of the horn torus, respectively. The unit circle $\{|z| = 1\}$ corresponds to the circle

$$\xi^2 + \eta^2 = \left(\frac{1}{2} \right)^2, \quad \zeta = \frac{1}{2}$$

in one to one way. Of course, the origin and the point at infinity are the same point and correspond to $(0, 0, 1/2)$. Furthermore, the inversion relation

$$z \longleftrightarrow \frac{1}{z}$$

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with respect to the unit circle $\{|z| = 1\}$ corresponds to the relation

$$(\xi, \eta, \zeta) \longleftrightarrow (\xi, \eta, 1 - \zeta)$$

and similarly,

$$z \longleftrightarrow -z$$

corresponds to the relation

$$(\xi, \eta, \zeta) \longleftrightarrow (-\xi, -\eta, \zeta)$$

and

$$z \longleftrightarrow -\frac{1}{\bar{z}}$$

corresponds to the relation

$$(\xi, \eta, \zeta) \longleftrightarrow (-\xi, -\eta, 1 - \zeta)$$

(H.G.W. Begehr: 2018.6.18.19:02).

We can see directly the important negative properties that the mapping is not isogonal (equiangular) and infinitely small circles do not correspond to infinitely small circles, as in analytic functions.

We note that only zero and numbers a of the form $|a| = 1$ have the property : $|a|^b = |a|, b \neq 0$. Here, note that we can also consider $0^b = 0$ ([4]). The symmetry of the horn torus model agrees perfectly with this fact. Only zero and numbers a of the form $|a| = 1$ correspond to points on the plane described by equation $\zeta - 1/2 = 0$. Only zero and numbers a of the form $|a| = 1$ correspond to points whose tangent lines to the surface of the horn torus are parallel to the axis ζ.

The horn torus should be considered as simply-connected ([2], 3585). We should consider that the origin and the point at infinity (that is represented by zero) is attached as one point on the $\mathbf{R}^3$ space. Certainly, a curve through the origin and the point at infinity is mapped to a closed curve on the horn torus and the closed curve can not be shrinked to a point, however, note that the point $(0, 0, 1/2)$ is a boundary point on the $\mathbf{R}^3$ space.

3 Conformal mapping from the plane to the horn torus with a modified mapping

W. W. Däumler discovered a surprising conformal mapping from the extended complex plane to the horn torus model (2018.8.18):

https://www.horntorus.com/manifolds/conformal.html
and
https://www.horntorus.com/manifolds/solution.html

Our situation is invariant by rotation around ζ axis, and so we shall consider the problem on the ξ, ζ plane.

Let $N(0,0,1)$ be the north pole. Let $P'(\xi, \eta, \zeta)$ denote a point on the Riemann sphere and let $z = x + iy$ be the common point with the line NP' and $\zeta = 0$ plane (: $z = x + iy$); that is P' is the stereographic projection map of the point $z = x + iy$ onto the unit sphere.

Let $M(1/4, 0, 1/2)$ be the center of the circle (1.1). Let P'' be the common point of the line SP' ($S = S(0, 0, 1/2)$) and the circle (1.1).

Let Q' be $(0, 0, \zeta)$ that is the line $Q'P'$ is parallel to the x axis. Let Q'' and M'' be the common points with the ζ axis and $\xi = 1/4$ with the parallel line to the x axis through the point P'', respectively.

Further, we set $\alpha = \angle OSP' = \angle P''IS = (1/2)\angle P''MS$ (I := I(1/2,0,1/2)). We set P for the point on the horn torus such that $\phi = \angle SMP$ and Q be the point on the ζ axis such that the line QP is parallel to the x axis.

Then, we have:
$$\overline{P'Q'} = \frac{1}{2}\sin\alpha,$$

$$\overline{P''M''} = \frac{1}{4}|\cos(2\alpha)|,$$

$$\overline{P''Q''} = \frac{1}{4}(1 - \cos(2\alpha)),$$

the length of latitude through P' is

$$2\pi\overline{P'Q'} = \pi\sin\alpha,$$

and the length of latitude through P''

$$2\pi\overline{P''Q''} = \frac{\pi}{2}(1 - \cos(2\alpha)) = \pi\sin^2\alpha.$$

Similarly, we have
$$2\pi\overline{QP} = \frac{\pi}{2}(1 - \cos\phi).$$

From the conformal mapping from the point P' to the point P, we have the identity
$$d\alpha : d\phi = \sin\alpha : 1 - \cos\phi;$$

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that is we have the differential equation

$$\frac{d\alpha}{\sin\alpha} = \frac{d\phi}{1-\cos\phi}.$$

Note here that the radius of the circle (1.1) is half of the stereographic projection mapping circle (the Riemann sphere). We solve this differential equation as, with an integral constant C

$$\log\mid\tan\frac{\alpha}{2}\mid = -\cot\frac{\phi}{2} + C.$$

For this derivation of the differential equation, see the detail comments in the site : **conformal mapping sphere $\leftrightarrow$ horn torus** with beautiful figures and many informations, by W. W. Däumler. In order to check his idea, we will give a complete proof analytically, in Section 5. As we refer to soon, W. W. Däumler considered various constants C, however, for the case $C = 0$, the situation is specially good as we see by numerical experiments, therefore, in this paper we will consider the case $C = 0$.

By solving for ϕ we have the result

$$\phi = 2\cot^{-1}(-\log\mid\tan(\alpha/2)\mid) \tag{3.1}$$

or

$$\alpha = 2\tan^{-1}(e^{(-\cot(\phi/2))}). \tag{3.2}$$

Next, note that

$$\tan\frac{\alpha}{2} = |z|$$

and

$$\alpha = 2\tan^{-1}|z|. \tag{3.3}$$

We thus have

$$\phi = 2\cot^{-1}(-\log|z|) \tag{3.4}$$

and the inverse is

$$|z| = e^{-\cot(\phi/2)}. \tag{3.5}$$

We thus obtain the complicated conformal mapping for the z plane to the horn torus by (3.4) and (3.2). The inverse conformal mapping for the horn torus to the complex z plane is given by (3.1) and (3.5).

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For the integral constant C, Däumler considers the general constant C and stated that:

I don't recognize a big problem with constant C. What are the crucial points? As I stated, all mappings from sphere to horn torus and inverse with any real C are conformal, but only the mappings with $C = 0$ are bijective. Respectively with

$$\alpha = 2 \tan^{-1}(p \cdot |z|)$$

and

$$|z| = \frac{\tan(\alpha/2)}{p},$$

all mappings from complex plane to sphere and inverse with real $p > 0$ are conformal, but bijective only when $p = 1$, what is the normal Riemannian stereographic projection. Main thing is to have at least one solution $(C = 0)$ in this topic, and we can keep other constants, C not equal 0 and p not equal 1, for special cases in different context.

For this very interesting topics, see his site.

We can represent the direct Däumler mapping from the z plane onto the horn torus as follows (V. V. Puha: 2018.8.28.22:31): With (3.4)

$$\xi = \frac{x \cdot (1/2)(\sin(\phi/2))^2}{\sqrt{x^2 + y^2}},$$

$$\eta = \frac{y \cdot (1/2)(\sin(\phi/2))^2}{\sqrt{x^2 + y^2}},$$

and

$$\zeta = -\frac{1}{4}\sin\phi + \frac{1}{2}.$$

Indeed, at first, we have

$$SP := L = 2 \cdot \frac{1}{4}\sin\frac{\phi}{2} = \frac{1}{2}\sin\frac{\phi}{2}, \tag{3.6}$$

$$\sqrt{\xi^2 + \eta^2} = L\cos\left(\frac{\pi}{2} - \frac{\phi}{2}\right) = L\sin\frac{\phi}{2},$$

and

$$\sqrt{\xi^2 + \eta^2} = \frac{1}{2}\sin^2\frac{\phi}{2}.$$

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From the simple relations

$$\xi = \frac{x\sqrt{\xi^2 + \eta^2}}{\sqrt{x^2 + y^2}}, \quad \eta = \frac{y\sqrt{\xi^2 + \eta^2}}{\sqrt{x^2 + y^2}}, \tag{3.7}$$

and

$$\zeta = -L\sin\left(\frac{\pi}{2} - \frac{\phi}{2}\right) + \frac{1}{2},$$

we have the desired representations.

We will give the inversion formula of the Däumler mapping. From (3.7) we have

$$x = \frac{\xi\sqrt{x^2 + y^2}}{\sqrt{\xi^2 + \eta^2}}, \quad y = \frac{\eta\sqrt{x^2 + y^2}}{\sqrt{\xi^2 + \eta^2}}.$$

Hence, it is enough to represent $\sqrt{x^2 + y^2}$ in terms of ξ, η, ζ on the horn torus. From (3.5), (3.6) and

$$L = \sqrt{\xi^2 + \eta^2 + \left(\zeta - \frac{1}{2}\right)^2},$$

we have the inversion formula from the horn torus to the x, y plane:

$$x = \frac{\xi}{\sqrt{\xi^2 + \eta^2}} \exp\pm\left\{\frac{\sqrt{\zeta - (\xi^2 + \eta^2 + \zeta^2)}}{\sqrt{\xi^2 + \eta^2 + \left(\zeta - \frac{1}{2}\right)^2}}\right\} \tag{3.8}$$

and

$$y = \frac{\eta}{\sqrt{\xi^2 + \eta^2}} \exp\pm\left\{\frac{\sqrt{\zeta - (\xi^2 + \eta^2 + \zeta^2)}}{\sqrt{\xi^2 + \eta^2 + \left(\zeta - \frac{1}{2}\right)^2}}\right\}. \tag{3.9}$$

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4 Properties of the Däumler conformal mapping

The Däumler conformal mapping stated in Section 3 is very complicated, however, has very beautiful properties. We will see its elementary properties.

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The circle $|z| = r$ is mapped to the circle:

$$\xi^2 + \eta^2 = \frac{1}{4}\left\{\sin\frac{\phi}{2}\right\}^4, \quad \zeta = -\frac{1}{4}\sin\phi + \frac{1}{2}$$

with

$$\frac{\phi}{2} = \cot^{-1}(-\log r).$$

In particular, note that the unit circle $r = 1$ is mapped to the circle

$$\xi^2 + \eta^2 = \left(\frac{1}{2}\right)^2, \quad \zeta = \frac{1}{2}.$$

We have the relation

$$\frac{\eta}{\xi} = \frac{y}{x},$$

but for $y = mx$

$$\zeta = -\frac{1}{4}\sin\left\{2\cot^{-1}\left(-\frac{1}{2}(\log x^2 + \log(1 + m^2))\right)\right\}.$$

Furthermore, the inversion relation

$$z \longleftrightarrow \frac{1}{\bar{z}}$$

with respect to the unit circle $\{|z| = 1\}$ corresponds to the relation

$$(\xi, \eta, \zeta) \longleftrightarrow (\xi, \eta, 1 - \zeta)$$

and similarly,

$$z \longleftrightarrow -z$$

corresponds to the relation

$$(\xi, \eta, \zeta) \longleftrightarrow (-\xi, -\eta, \zeta)$$

and

$$z \longleftrightarrow -\frac{1}{\bar{z}}$$

corresponds to the relation

$$(\xi, \eta, \zeta) \longleftrightarrow (-\xi, -\eta, 1 - \zeta).$$

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Of course, the conformal mapping of Däumler is important, however, its mapping is very involved and the difference with the Puha mapping in Section 2 is just the shift on the circle of longitude and the Puha mapping is very simple. Furthermore the Puha mapping is clear in the geometrical correspondence. Therefore, we will be able to enjoy the Puha mapping for the horn torus model.

5 Proof of conformal mapping

In order to confirm the conformality of the Däumler mapping and at the same time, in order to see its analytical structure, we will examine it. In this section, for simplicity we use L, N with $L = \log(x^2+y^2)$ and $N = L/(4+L^2)$.

First, we calculate the first order derivatives.

$$\frac{\partial \xi}{\partial x} = \frac{8y^2 - 8x^2L + 2y^2L^2}{(x^2 + y^2)^{3/2}(4 + L^2)^2},$$

$$\frac{\partial \xi}{\partial y} = \frac{\partial \eta}{\partial x} = \frac{-2xy(2 + L)^2}{(x^2 + y^2)^{3/2}(4 + L^2)^2},$$

$$\frac{\partial \eta}{\partial y} = \frac{2(4x^2 - 4y^2L + x^2L^2)}{(x^2 + y^2)^{3/2}(4 + L^2)^2},$$

$$\frac{\partial \zeta}{\partial x} = \frac{-2x(-4 + L^2)}{(x^2 + y^2)(4 + L^2)^2}$$

and

$$\frac{\partial \zeta}{\partial y} = \frac{-2y(-4 + L^2)}{(x^2 + y^2)(4 + L^2)^2}.$$

Next, we wish to have the relation between

$$(d\sigma)^2 = (d\xi)^2 + (d\eta)^2 + (d\zeta)^2$$

and

$$(ds)^2 = (dx)^2 + (dy)^2.$$

From

$$d\xi = \frac{2(-xydy(2+L)^2 + dx(4y^2 - 4x^2L + y^2L^2))}{(x^2+y^2)^{3/2}(4+L^2)^2},$$

$$d\eta = \frac{2(-xydx(2+L)^2 + dy(4x^2 - 4y^2L + x^2L^2))}{(x^2+y^2)^{3/2}(4+L^2)^2},$$

and

$$d\zeta = \frac{-2(xdx + ydy)(-4+L^2)}{(x^2+y^2)(4+L^2)^2},$$

we obtain the beautiful identity

$$(d\sigma)^2 = \frac{4(ds)^2}{(x^2+y^2)(4+L^2)^2}. \tag{5.1}$$

The next and final crucial point is the relation:

$$\frac{dx}{ds}, \frac{dy}{ds}$$

and

$$\frac{d\xi}{d\sigma}, \frac{d\eta}{d\sigma}, \frac{d\zeta}{d\sigma}.$$

This may be done directly by division by $d\sigma$ in (5.1). Indeed, we have:

$$\frac{d\xi}{d\sigma} = \frac{dx(y^2 - 4x^2N) + dy(-xy - 4xyN)}{ds(x^2+y^2)}, \tag{5.2}$$

$$\frac{d\eta}{d\sigma} = \frac{dx(-xy - 4xyN) + dy(x^2 - 4y^2N))}{ds(x^2+y^2)}, \tag{5.3}$$

and

$$\frac{d\zeta}{d\sigma} = \frac{-xdx(-4+L^2) - ydy(-4+L^2)}{ds\sqrt{x^2+y^2}(4+L^2)}. \tag{5.4}$$

On a point $P_0(\xi_0, \eta_0, \zeta_0)$ on the horn torus we consider two smooth curves passing the point

$$f_j(\xi, \eta, \zeta) = 0, \quad j = 1, 2.$$

At the point P_0, we denote the values of $\frac{d\xi}{d\sigma}, \frac{d\eta}{d\sigma}, \frac{d\zeta}{d\sigma}$ by λ_j, μ_j, ν_j, respectively. Then, for the angle Φ made by the curves at the point P_0 we have

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$$\cos \Phi = \lambda_1 \lambda_2 + \mu_1 \mu_2 + \nu_1 \nu_2. \tag{5.5}$$

The corresponding relations on the x, y plane are as follows:

For the corresponding curves on the x, y plane

$$g_j(x, y) = 0, \quad j = 1, 2,$$

at the corresponding point $Q_0(x_0, y_0)$, we denote the values of $\frac{dx}{ds}, \frac{dy}{ds}$ by α_j, β_j, respectively. Then, for the angle ϕ of the curves at the point $Q_0(x_0, y_0)$ we have

$$\cos \phi = \alpha_1 \alpha_2 + \beta_1 \beta_2. \tag{5.6}$$

We wish to prove that $(5.5) = (5.6)$, by formal calculation.

Note that from (5.2), we have for $(x, y) = (x_0, y_0)$, here, for simplicity we shall use (x, y) at Q_0

$$\lambda_j = \frac{\alpha_j(y^2 - 4x^2 N) + \beta_j(-xy - 4xyN)}{x^2 + y^2}.$$

Similarly, from (5.3),

$$\mu_j = \frac{\alpha_j(-xy - 4xyN) + \beta_j(x^2 - 4y^2 N)}{x^2 + y^2},$$

and from (5.4),

$$\nu_j = \frac{-x\alpha_j(-4 + L^2) - y\beta_j(-4 + L^2)}{\sqrt{x^2 + y^2}(4 + L^2)}.$$

When we insert these in the right side of (5.5), we obtain the right side of (5.6). In this section, for these formal calculations, we used MATHEMATICA.

6 Division by zero calculus

The division by zero with mysterious and long history was indeed trivial and clear as

$$\frac{1}{0} = \frac{0}{0} = \frac{z}{0} = \tan \frac{\pi}{2} = 0$$

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in the sense of generalized fractions and by the division by zero calculus by means of the Laurent expansion for analytic functions. See the papers [3], [7] and [8].

For the fundamental function $W = 1/z$ we did not consider any value at the origin $z = 0$, because we did not consider the division by zero $1/0$ in a good way. Many and many people consider its value by the limiting like $+\infty$ and $-\infty$ or the point at infinity as ∞. However, their basic idea comes from **continuity** with the common sense or based on the basic idea of Aristotle. However, as the division by zero we will consider its value of the function $W = 1/z$ as zero at $z = 0$. We will see that this new definition is valid widely in mathematics and mathematical sciences, see ([5, 6]) for example. Therefore, the division by zero will give great impacts to calculus, Euclidean geometry, analytic geometry, complex analysis and the theory of differential equations at an undergraduate level and furthermore to our basic ideas for the space and universe.

For the extended complex plane, we consider its stereographic projection mapping as the Riemann sphere and the point at infinity is realized as the north pole in the Alexsandroff's one point compactification. The Riemann sphere model gives a beautiful and complete realization of the extended complex plane through the stereographic projection mapping and the mapping has beautiful properties like isogonal (equiangular) and circle to circle correspondence (circle transformation). Therefore, the Riemann sphere is a very classical concept [1].

Now, with the division by zero we have to admit the strong discontinuity at the point at infinity, because the point at infinity is represented by zero. In [6], a formal contradiction for the classical result $1/0 = \infty$ was given and the strong discontinuity was shown in many and many examples. See [7] for example. On this situation, the third author discovered the mapping of the extended complex plane to a beautiful horn torus at (2018.6.4.7:22) and its inverse at (2018.6.18.22:18).

Incidentally, independently of the division by zero, W. W. Däumler has various special great ideas on horn torus as we see from his site:

Horn Torus & Physics (https://www.horntorus.com/) 'Geometry Of Everything', intellectual game to reveal engrams of dimensional thinking and proposal for a different approach to physical questions ...

In the horn torus model, we can find many division by zero properties.

In (1.2), we see the division by zero

$$0 = \frac{0}{0}$$

that shows the mapping from $(0, 0, 1)$ to $(0, 0)$.

In (1.3) and (1.4), we can see the same division by zero that shows the mapping from $(0, 0, 1/2)$ to $(0, 0)$.

Using the correspondence

$$\alpha = 0 \leftrightarrow \phi = 0,$$

or

$$\alpha = \pi/2 \leftrightarrow \phi = \pi$$

or

$$\alpha = \pi \leftrightarrow \phi = 2\pi,$$

we have $C = 0$ in the general Däumler mapping. Note that $\tan(\pi/2) = 0$, $\cot(\pi/2) = 0$ and $\log 0 = 0$ ([4]).

7   Acknowledgements

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